OCR
Oxford Cambridge and RSA

# Tuesday 20 June 2017 – Afternoon

## A2 GCE MATHEMATICS

**4735/01** Probability & Statistics 4

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4735/01
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

**1** A meteorologist claims that the median daily rainfall in London is 2.2 mm. A single sample sign test is to be used to test the claim, using the following hypotheses:

$H_0$: a sample comes from a population with median 2.2,

$H_1$: the sample does not come from a population with median 2.2.

30 randomly selected observations of daily rainfall in London are compared with 2.2, and given a '+' sign if greater than 2.2 and a '–' sign if less than 2.2. (You may assume that no data values are exactly equal to 2.2.) The test is to be carried out at the 5% level of significance. Let the number of '+' signs be $k$. Find, in terms of $k$, the critical region for the test showing the values of any relevant probabilities. **[4]**

**2** The independent discrete random variables $X$ and $Y$ can take the values 0, 1 and 2 with probabilities as given in the tables.

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X = x)$ | 0.5 | 0.3 | 0.2 |

| $y$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(Y = y)$ | 0.5 | 0.3 | 0.2 |

The random variables $U$ and $V$ are defined as follows:

$$U = XY, V = |X - Y|.$$

**(i)** In the Printed Answer Book complete the table giving the joint distribution of $U$ and $V$. **[4]**

**(ii)** Find Cov ($U$, $V$). **[5]**

**(iii)** Find $P(UV = 0 | V = 2)$. **[2]**

**3** For events $A$, $B$ and $C$ it is given that $P(A) = 0.6$, $P(B) = 0.5$, $P(C) = 0.4$ and $P(A \cap B \cap C) = 0.1$. It is also given that events $A$ and $B$ are independent and that events $A$ and $C$ are independent.

**(i)** Find $P(B | A)$. **[1]**

**(ii)** Given also that events $B$ and $C$ are independent, find $P(A' \cap B' \cap C')$. **[4]**

**(iii)** Given instead that events $B$ and $C$ are **not** independent, find the greatest and least possible values of $P(A' \cap B' \cap C')$. **[5]**

**4** The heights of eleven randomly selected primary school children are measured. The results, in metres, are

Girls 1.48 1.31 1.63 1.38 1.56 1.57

Boys 1.44 1.35 1.32 1.28 1.27.

**(i)** Use a Wilcoxon rank-sum test, at the 1% significance level, to test whether primary school girls are taller than primary school boys. **[6]**

**(ii)** It is decided to repeat the test, using larger random samples. The heights of twenty girls and eighteen boys are measured. Find the greatest value of the test statistic $W$ which will result in the conclusion that there is evidence, at the 1% level of significance, that primary school girls are taller than primary school boys. **[6]**

**5** The discrete random variable $X$ is such that $\mathrm{P}(X = x) = \frac{3}{4}\left(\frac{1}{4}\right)^x$, $x = 0, 1, 2, \ldots$ .

**(i)** Show that the moment generating function of $X$, $\mathrm{M}_X(t)$, can be written as $\mathrm{M}_X(t) = \frac{3}{4 - \mathrm{e}^t}$. **[4]**

**(ii)** Find the range of values of $t$ for which the formula for $\mathrm{M}_X(t)$ in part **(i)** is valid. **[2]**

**(iii)** Use $\mathrm{M}_X(t)$ to find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[5]**

**6** The continuous random variable $Z$ has probability density function

$$\mathrm{f}(z) = \begin{cases} \frac{4z^3}{k^4} & 0 \leqslant z \leqslant k, \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ is a parameter whose value is to be estimated.

**(i)** Show that $\frac{5Z}{4}$ is an unbiased estimator of $k$. **[4]**

**(ii)** Find the variance of $\frac{5Z}{4}$. **[5]**

The parameter $k$ can also be estimated by making observations of a random variable $X$ which has mean $\frac{1}{2}k$ and variance $\frac{1}{12}k^2$. Let $Y = X_1 + X_2 + X_3$ where $X_1$, $X_2$ and $X_3$ are independent observations of $X$.

**(iii)** $cY$ is also an unbiased estimator of $k$. Find the value of $c$. **[2]**

**(iv)** For the value of $c$ found in part **(iii)**, determine which of $\frac{5Z}{4}$ and $cY$ is the more efficient estimator of $k$. **[4]**

**7** The discrete random variable $Y$ has probability generating function $\mathrm{G}_Y(t) = \frac{1}{126}t(64 - t^6)\left(1 - \frac{t}{2}\right)^{-1}$.

**(i)** Find $\mathrm{P}(Y = 3)$. **[5]**

**(ii)** Find $\mathrm{E}(Y)$. **[4]**

**END OF QUESTION PAPER**